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Fate of Z_N walls in hot holographic QCD

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Abstract: We first study Z_N walls/interfaces in a deconfined phase of Witten's $D4$-brane background of pure SU(N) Yang-Mills theory, motivated by a recent work in the case of $N = 4$ SYM. Similarly to it, we propose that for a large wall charge $k \sim N$, it is described by k $D2$-branes blown up into a $NS5$-brane wrapping S^3 inside S^4 via Myers effect, and we calculate the tension by suitable U-duality. We find a precise Casimir scaling for the tension formula. We then study the fate of Z_N-vacua in a presence of fundamental flavors in quenched approximation via gauge/gravity correspondence. In the case of $D3/D7$ system where one can vary the mass m_q of flavors, we show that there is a phase transition at $T_c \sim m_q$, below which the Z_N-vacua survive while they are lifted above the critical temperature. We analytically calculate the energy lift of k'th vacua in the massless case, both in the $D3/D7$ system and in the Sakai-Sugimoto model.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence

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1 Introduction

Gauge/gravity correspondence has often provided us easy tools for addressing difficult problems of strongly interacting gauge theories. It is based on the observation that large N, large t'Hooft coupling limit of certain gauge theories have alternative weakly-coupled effective descriptions in terms of gravity or string theory [1]. One important aspect of these dual theories is that the spacetime they are living is one-dimension higher than the original spacetime of gauge theories, and the additional dimension, often referred to as holographic coordinate, plays a physical role of energy scale of a given process. This mapping of space-energy is practically achieved by the presence of a warping factor in the metric that depends on the additional dimension, which provides a geometrical potential along the holographic dimension.

Out of vast amount of previous works that have been done over a decade to test and study this idea, one interesting application has been to describing the deconfined phase of gauge theories in a sufficiently high temperature, whose dual picture is a black-hole spacetime with its horizon located at some point in the holographic coordinate [2, 3]. Due to the geometric barrier along holographic dimension provided by the warping factor in the metric, this black-hole spacetime is stable against its Hawking radiation, resulting in an equilibrium state called Hartle-Hawking state. Performing relatively easy analyses in this black-hole background has given us many valuable, typically non-trivial, results about physics of high temperature, deconfined gauge theories in strong coupling [4].

In this work, we would like to add one more piece to the mounted pile of results about high temperature phase of gauge theories obtained using gauge/gravity correspondence. Our motivation is two-fold. We first study Z_N walls in the Euclidean picture of finite

temperature deconfined phase of pure SU(N) Yang-Mills theory using the Witten's dual background of $D4$-branes [3]. We are interested in the k-wall with $\frac{k}{N} \sim \mathcal{O}(1)$, which is largely motivated by a recent work of Armoni, Kumar and Ridgway for the case of $N = 4$ SYM via the $AdS_5 \times S^5$ background [5]. Similar to their result, we will show that the k-wall with $\frac{k}{N} \sim \mathcal{O}(1)$ is k (Euclidean) $D2$-branes blown up to a single $NS5$-brane wrapping S^3 inside the S^4 in the background via Myers effect. We calculate its tension after performing suitable U-dualities of Type II SUGRA to make the problem more tractable. Our result of k-wall tension indicates certain limitations of finite temperature black-hole geometry of Witten's background that has been used in the previous literature.

Our second motivation is to study the fate of Z_N discrete vacua, and hence Z_N walls connecting them, when we include fundamental flavor quarks in the gauge theory. One naturally expects that since Z_N is no longer a symmetry of the theory in the presence of fundamental matters, these Z_N vacua are generally lifted except the one true vacuum. However, there appears an issue when the fundamental matters are much more massive than the temperature, because the physics with energy scale below the mass of fundamental matters decouples from the fundamental matters, and the effective low energy theory would still have a center Z_N symmetry. One logical possibility is that when $\frac{m_q}{T} \gg 1$, the Z_N vacua and domain walls persist in the theory, while they disappear at $\frac{m_q}{T} \ll 1$, and these two phases are separated by a phase transition at $\frac{m_q}{T_c} \sim \mathcal{O}(1)$. We will give an evidence to this scenario in the framework of gauge/gravity correspondence using $D3/D7$-brane system corresponding to $N = 2$ fundamental hypermultiplets in $N = 4$ SYM [6].

In the case of massless fundamental flavors, Z_N vacua are lifted unambiguously. Using gauge/gravity correspondence, we calculate analytically this energy lift of k-vacua in quenched approximation, in both $D3/D7$ system and in the Sakai-Sugimoto model [7]. As a by-product, we also find an infinite tower of stable states above each lifted k-vacua, similar to the tower of stable states above θ-vacuum that was found in ref. [8].

Note added in revision. The author was informed of an issue of correct interpretation of center Z_N symmetry and the corresponding Z_N walls in Euclidean thermal SU(N) gauge theory.[1] If one takes SU(N)/Z_N as the gauge group rather than SU(N), the center Z_N symmetry is absent and the Z_N walls are not the Minkowski domain walls, but instead some Euclidean instanton-like objects [9]. In the presence of fundamental flavors, the gauge group is indeed SU(N) and the center Z_N symmetry can be at least an approximate symmetry of the theory. Our second subject of the paper should be viewed in this way. Alternatively, one can think of thermal S^1 as a spatial compactification, and consider 3-dimensional Minkowski theory after Wick rotation along one non-compact spatial direction. In that context, one can talk about Minkowski domain walls in the 3 dimensional theory.

2 Review of center Z_N in gauge/gravity correspondence

This section is devoted to a brief review on the center Z_N symmetry of the Euclidean description of finite temperature SU(N) gauge theory with only adjoint matters, and its

[1] We thank Andrei Smilga for bringing this issue to our attention.

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spontaneous breaking in the deconfined phase via non-vanishing Polyakov line along the thermal circle [10, 11]. We will also summarize relevant results in refs. [5, 12] in the context of gauge/gravity correspondence to lay ground for our analyses in subsequent sections. Readers familiar to these subjects can jump into the next section.

2.1 Center Z_N and its spontaneous breaking in hot SU(N) gauge theory

An Euclidean finite temperature gauge theory is defined in the space $S^1 \times R^3$, where the thermal circle, or Euclidean time, has a period $\beta = \frac{1}{T}$. As a SU(N) gauge theory, field configurations that are connected by SU(N) gauge transformations must be treated as identical to each other, and one should mod out the space of fields by gauge transformations. Having S^1 means that the local gauge function $\mathrm{U}(x) \in \mathrm{SU}(N)$ that we mod out should be periodic along S^1 to preserve usual periodicity of matter fields that are charged under SU(N). If the theory contains only adjoint matters, one might think of an extended notion of gauge transformation whose gauge function $\mathrm{U}(x) \in \mathrm{SU}(N)$ is periodic only up to an element of the center Z_N inside SU(N). This is because for adjoint matters Φ, as Z_N freely commutes with adjoint Φ, the gauge transformation

$$\Phi(x) \to \mathrm{U}(x)^\dagger \Phi(x) \mathrm{U}(x), \tag{2.1}$$

doesn't change the periodicity of Φ, and seems to be allowed without any problem. Note however that under the extended gauge transformation, the Wilson line along the thermal S^1, called Polyakov line,

$$W(S^1) = \frac{1}{N} \mathrm{Tr} P \exp\left(i \int_0^\beta A_\tau d\tau \right), \tag{2.2}$$

transforms exactly by the center Z_N element $W(S^1) \to e^{\frac{2\pi i k}{N}} W(S^1)$. Therefore, if we accept $W(S^1)$ as one of physical "gauge invariant" observables of the theory, the extended gauge transformations are in fact not allowed as gauge transformations that we mod out. Rather they should be thought of as symmetries of the theory as the action is clearly invariant under the transformations. The extended gauge transformations are constant Z_N times the usual periodic gauge transformations that we mod out, so that the resulting symmetry is a global Z_N symmetry.

One can immediately identify $W(S^1)$ as an order parameter of the spontaneous breaking of this center Z_N symmetry. In fact, it can be argued to get a non-vanishing expectation value in the deconfined phase of high temperature, while it should vanish in the confined phase. The rough picture is that $W(S^1)$ represents a world-line of an external fundamental quarks sitting at a point in R^3 in Euclidean finite temperature description. In the confined phase, its presence would cost too much free energy due to confinement so that the partition function with $W(S^1)$ would vanish, while one generally expects a finite non-vanishing result in the deconfined phase. Therefore, Z_N is spontaneously broken in the deconfined phase, and there exist N-number of vacua parameterized by the Z_N phase of the order parameter $W(S^1)$. One naturally thinks of domain walls connecting these vacua. By k-wall, we will refer to a domain wall connecting i'th vacuum and $i+k$'th vacuum, and k is identified with $k+N$ by definition.

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There is a beautiful connection between a Z_N domain wall and a large spatial t'Hooft line [13]. Consider a large spatial t'Hooft line along a curve C which bounds a large surface S spanning two space directions in R^3, say x^1 and x^2. By t'Hooft line, we mean inserting a magnetic monopole whose world-line is the curve C. In the presence of a magnetic monopole, the Bianchi identity for the electric gauge potential A is violated by

$$dF = \delta_C^{(3)} \,, \tag{2.3}$$

where $\delta_C^{(3)}$ is the Poincare dual 3-form to C. Being familiar to the case of Dirac monopole, we can think of S as a Dirac sheet, a world-sheet spanned by a Dirac string. As $\partial S = C$, we have

$$d\delta_S^{(2)} = \delta_C^{(3)} \,, \tag{2.4}$$

and we see that

$$F = dA = \delta_S^{(2)} \,. \tag{2.5}$$

Then, consider a cylinder D made of S^1 times a finite interval in R^3 which connects two points P, Q from the two opposite sides of the surface S, so that this interval crosses S at a single point in R^3. It is clear that this cylinder has an intersection number 1 with the surface S in our spacetime $S^1 \times R^3$, or equivalently

$$\int_D \delta_S^{(2)} = \#(D, S) = 1 \,. \tag{2.6}$$

Using (2.5), the left-hand side in the above is

$$\int_D F = \int_{\partial D} A = \int_{S^1 atP} A - \int_{S^1 atQ} A \,, \tag{2.7}$$

which tells us that the phase of Polyakov line jumps across the surface S of Dirac sheet, and we can think of S as a domain wall separating regions of different Polyakov lines. Although the above discussion is given in terms of an Abelian gauge theory, the logic is essentially identical in the case of non-Abelian SU(N) gauge theory, and the conclusion is that the k-wall can be thought of as the minimal surface S bounded by a large spatial t'Hooft line of k-th anti-symmetric representation of the magnetic group.

2.2 Z_N vacua and Z_N walls in gauge/gravity correspondence

As one typically considers the large N limit in discussing gauge/gravity correspondence, it may at first seem unlikely to see discrete Z_N symmetry in the gravity dual description. However, believing AdS/CFT correspondence beyond the leading large N limit, one should be able to access sub-leading $\frac{1}{N}$ effects by carefully taking into account quantum effects in the gravity side, which is not easy in general. In the case of Z_N, Aharony and Witten successfully identified the relevant quantum effect in $AdS_5 \times S^5$ background, that is responsible for the discrete Z_N symmetry appearing in the gravity dual description of the $N = 4$ SYM [12].

For our purposes, we will recapitulate only the case of Poincare patch version of $AdS_5 \times S^5$ corresponding to the gauge theory defined on $S^1 \times R^3$, while we refer the readers to

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ref. [12] for the discussion of $N = 4$ SYM theory on $S^1 \times S^3$. This is because only in the former case, we can have a spontaneous breaking of global symmetry such as Z_N that we are interested in. A finite temperature phase of $N = 4$ SYM in Euclidean description has a unique dual geometry given by the Euclideanized black-hole in the Poincare patch;

$$ds_E^2 = \frac{r^2}{L^2}\left[\left(1 - \frac{\pi^4 T^4 L^8}{r^4}\right) dt_E^2 + dx_i^2\right] + \frac{L^2}{r^2}\frac{1}{\left(1 - \frac{\pi^4 T^4 L^8}{r^4}\right)} dr^2 + L^2 d\Omega_5^2 \,, \tag{2.8}$$

where the thermal circle combined with the holographic radial coordinate r makes a two-dimensional cigar shape D that closes off at the location of the horizon $r_H = \pi T L^2$ with $L^4 = 4\pi g_s N l_s^4$. The theory is always in the deconfined phase due to conformal symmetry. As is well-known, the expectation value of a Wilson line is calculated by the semi-classical string world-sheet which has a boundary at UV $r \to \infty$ along the curve of the Wilson line [14, 15]. One easily identifies such string world-sheet in the case of our Polyakov temporal Wilson line $W(S^1)$; it spans precisely the cigar D of the thermal circle and r. Because the world-sheet closes off at a finite point $r = r_H$, it would give us a finite value of the string action, and hence non-vanishing expectation value of $W(S^1)$ after suitable holographic renormalization [16], which is in accord with the fact that $W(S^1)$ is non-vanishing in a deconfined phase. However, there appears a puzzle since it seems one can freely turn on NS 2-form B with $dB = 0$ on the cigar without any change in the Type IIB SUGRA action, with arbitrary values of

$$\frac{1}{2\pi l_s^2}\int_D B \,, \tag{2.9}$$

which appears as a phase of our semi-classical string amplitude, and hence of $W(S^1)$. As this B mode is a normalizable mode, we in fact have to sum over these possible phases in the partition function, which would make the expectation value of $W(S^1)$ zero [3]. To prevent this, there must be a mechanism that lifts the degeneracy among continuous values of NS B field, and we indeed also need this to have Z_N discrete values of the phase of $W(S^1)$.

Ref. [12] showed that the necessary mechanism is from quantization of the RR 2-form C in our background with N D3 flux on S^5,

$$\frac{1}{(2\pi l_s)^4}\int_{S^5} F_5 = N \,, \tag{2.10}$$

such that the action contains the piece

$$N \cdot \left(\frac{1}{2\pi l_s^2}\int_D B\right) \cdot \frac{1}{(2\pi l_s)^2}\int_{R^3} dC \equiv \theta \cdot \frac{1}{(2\pi l_s)^2}\int_{R^3} dC \,, \tag{2.11}$$

from the Type IIB term $\frac{2\pi}{(2\pi l_s)^8}\int_{M_{10}} F_5 \wedge B_2 \wedge H_3$ with $H = dC$. With the usual kinetic term for $H = dC$ along R^3 obtained after integrating out $S^5 \times D$,

$$\frac{g_s N}{2^7 \pi^5 l_s^4 T^3}\int_{R^3} (H_{123})^2 \equiv \frac{1}{2e^2}\int_{R^3} (H_{123})^2 \,, \tag{2.12}$$

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the above θ was shown to play a similar role as the $\frac{\theta}{2\pi}F_{01}$ term in 2D Abelian U(1) theory, where the vacuum energy is given by

$$E_\theta = \frac{e^2}{2(2\pi l_s)^4} \cdot \min_{k\in Z}(2\pi k - \theta)^2 \,, \tag{2.13}$$

with integer k labelling certain allowed quantum states of the system. This says $\theta = 2\pi Z$ are discrete vacua among continuous possibilities, and looking back (2.11), one readily finds discrete Z_N vacua of

$$\frac{1}{2\pi l_s^2}\int_D B = \frac{2\pi k}{N} \,, \qquad k \sim k + N \,, \tag{2.14}$$

in the quantum theory, which at the same time also explains the Z_N phase of $W(S^1)$ at each vacua.

The Z_N walls in the gravity description can most easily be identified by considering their connection to large spatial t'Hooft lines that we discussed above. In the simplest case of $k = 1$ wall, the t'Hooft line corresponds to a (Euclidean) $D1$ brane world-sheet whose boundary at $r \to \infty$ is along the t'Hooft line curve. One may think of the $D1$ brane as the Dirac sheet of the t'Hooft line, and one concludes that $D1$ brane spanning along two spatial directions in R^3 is the $k = 1$ domain wall. Note that when the t'Hooft line is large, most of the $D1$ world-sheet will sit at the IR end $r = r_H$ to minimize its tension. Therefore, the $D1$ brane sitting at $r = r_H$ and a point in S^5, while spanning two directions in R^3 is the final configuration we are seeking for the $k = 1$ wall in $N = 4$ SYM. In a recent work by Armoni, Kumar and Ridgway [5], they proposed that for k-walls with $\frac{k}{N} \sim \mathcal{O}(1)$, the k $D1$ branes are blown up to a $NS5$ brane wrapping $S^4 \subset S^5$ via Myers effect, and the result for the k-wall tension in their picture supports this claim, that is, for low k the tension is simply that of the k $D1$ branes, and $(N - k)$-wall tension is equal to the k-wall tension.

3 *k*-walls in the Witten's *D*4-brane background

We come to one of our objectives in this work; calculating k-wall tension with $\frac{k}{N} \sim \mathcal{O}(1)$ in the (approximate) gravity dual of pure SU(N) Yang-Mills theory. We first discuss briefly how center Z_N appears in the Witten's background, which is rather closely parallel to the case of $AdS_5 \times S^5$.

There are two finite temperature Euclidean geometries competing with each other near the confinement-deconfinement phase transition; a confined phase at low T where the thermal circle remains finite in all spacetime region, and a deconfined phase of Euclideanized black-hole where the thermal circle shrinks to zero at the horizon, making a cigar shape combined with the holographic direction [17].

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The confined phase geometry is

$$
\begin{aligned}
ds^2 &= \left(\frac{U}{R}\right)^{\frac{3}{2}}\left(dt_E^2+dx_i^2+f(U)dx_4^2\right)+\left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)}+U^2d\Omega_4^2\right),\\
F_4 &= \frac{(2\pi l_s)^3N}{V_4}\epsilon_4, \qquad e^{\phi}=g_s\left(\frac{U}{R}\right)^{\frac{3}{4}}, \quad V_4=\mathrm{Vol}(S^4)=\frac{8\pi^2}{3},\\
R^3 &= \pi g_s N l_s^3, \qquad f(U)=1-\left(\frac{U_{KK}}{U}\right)^3,
\end{aligned}
\tag{3.1}
$$

with the thermal circle t_E periodic $t_E \sim t_E+\beta$, and x_4 is compactified with the period

$$
\delta x_4 = \frac{4\pi}{3}\left(\frac{R^3}{U_{KK}}\right)^{\frac{1}{2}} \equiv \frac{2\pi}{M_{KK}}\,. \tag{3.2}
$$

The useful relation between the above parameters and the 4D gauge theory data on (t_E, x_i) is given by [7]

$$
R^3=\frac{g_{\rm YM}^2Nl_s^2}{2M_{KK}}\,, \quad U_{KK}=\frac{2}{9}g_{\rm YM}^2NM_{KK}l_s^2\,, \quad g_s=\frac{g_{\rm YM}^2}{2\pi M_{KK}l_s}\,. \tag{3.3}
$$

As usual the Polyakov line expectation value is described by a string world-sheet wrapping t_E and one more direction in the geometry, and since t_E never shrinks to zero one easily finds that there is no way this string stops in the geometry without turning back to the UV boundary $U\to\infty$, which corresponds to another anti-Polyakov line in the gauge theory. In short, $\langle W(S^1)\rangle = 0$ with a single insertion of $W(S^1)$ in the gauge theory, in accord to the expectation in a confined phase. The way discrete Z_N symmetry appears in the situation is that if we insert $W(S^1)$'s by multiple N times, these multiple N number of string world-sheets can now sit on (Euclidean) $D4$ branes wrapping $S^4\times t_E$, because on this baryonic $D4$-brane, there is a Chern-Simons coupling

$$
\mu_4\int_{D4}F_4\wedge(2\pi l_s^2)A=\frac{1}{(2\pi l_s)^3}\int_{D4}F_4\wedge A=N\int dt_E\,A_0\,, \tag{3.4}
$$

which induces N string charges on the world-volume, so that N strings can sit on it [18]. Therefore, multiple N times insertion of $W(S^1)$'s gives us a finite expectation value and one concludes that there is a Z_N symmetry under which $W(S^1)$ has a unit charge. Note that the Z_N symmetry is unbroken in this phase due to $\langle W(S^1)\rangle = 0$.

Our present interest concerns more about the deconfined phase at high T, for which the dual geometry is given by

$$
\begin{aligned}
ds^2 &= \left(\frac{U}{R}\right)^{\frac{3}{2}}\left(f(U)dt_E^2+dx_i^2+\frac{1}{(M_{KK}l_s)^2}dx_4^2\right)+\left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)}+U^2d\Omega_4^2\right),\\
F_4 &= \frac{(2\pi l_s)^3N}{V_4}\epsilon_4\,, \qquad e^{\phi}=g_s\left(\frac{U}{R}\right)^{\frac{3}{4}}, \qquad V_4=\mathrm{Vol}(S^4)=\frac{8\pi^2}{3}\,,\\
R^3 &= \pi g_s N l_s^3\,, \qquad f(U)=1-\left(\frac{U_T}{U}\right)^3,
\end{aligned}
\tag{3.5}
$$

where the period of the thermal circle, $\beta = \frac{1}{T}$, is related to U_T by

$$\beta = \frac{4\pi}{3}\left(\frac{R^3}{U_T}\right)^{\frac{1}{2}} , \tag{3.6}$$

and we have re-scaled x_4 such that its period is $(2\pi l_s)$ for later convenience. In this background, the string world-sheet for a Polyakov line can stop at $U = U_T$ where the thermal circle t_E shrinks to zero making a cigar shape D with $U \geq U_T$, so that the string action becomes finite and one expects a non-vanishing VEV of $W(S^1)$. The same issue of NS 2-form phase $\frac{1}{2\pi l_s^2}\int_D B^{NS}$ arises as in the case of $AdS_5 \times S^5$, and the resolution is also similar. In Type IIA, there is a term

$$\frac{2\pi}{(2\pi l_s)^8}\int_{M^{10}} F_4 \wedge F_4 \wedge B_2 = N \cdot \left(\frac{1}{2\pi l_s^2}\int_D B^{NS}\right)\frac{1}{(2\pi l_s)^3}\int_{x_i,x_4} F_4 \equiv \theta \cdot \frac{1}{(2\pi l_s)^3}\int_{x_i,x_4} F_4 , \tag{3.7}$$

and considering quantizing F_4^{RR} along (x_i, x_4) after integrating over $S^4 \times D$ to get the kinetic term

$$\frac{3g_s N}{2^9\pi^6 l_s^5 T^3}\int dx_i dx_4 \ (F_{1234})^2 , \tag{3.8}$$

we have the integer parameterized vacua of $\theta = 2\pi Z$ to minimize the energy of the allowed quantum states treating x_1 as a time,

$$\Psi_k = \exp\left(ik\mu_2\int_{x_{2,3,4}} C_3^{RR}\right) = \exp\left(\frac{2\pi ik}{(2\pi l_s)^3}\int_{x_{2,3,4}} C_3^{RR}\right) . \tag{3.9}$$

This provides us the Z_N vacua of

$$\frac{1}{2\pi l_s^2}\int_D B^{NS} = \frac{2\pi k}{N} , \qquad k \sim k + N , \tag{3.10}$$

corresponding to the spontaneous breaking of Z_N by $\langle W(S^1)\rangle \neq 0$.

It is not difficult to identify the string theory object that plays a role of Z_N domain walls, at least for k-walls with sufficiently low k's. Again, using the large spatial t'Hooft line is most convenient. In our $D4$ background, the magnetically charged sources compared to the electrical degrees of freedom of fundamental strings are provided by x_4 wrapping $D2$-branes [19]. This can also be seen by considering T-duality along x_4 upon which N $D4$-branes become $D3$-branes and $D2$ becomes $D1$, the usual magnetic degrees of freedom. Consequently, a large spatial t'Hooft line will be the boundary of the world-volume of a x_4-wrapped (Euclidean) $D2$-brane at $U \to \infty$, and most of the $D2$-brane world-volume will sit at the IR end $U = U_T$ to minimize its tension. We can think of two spatial directions in R^3 that $D2$-brane is spanning as the Dirac sheet corresponding to the t'Hooft line, and hence the $k = 1$ domain wall. Therefore, the $D2$-brane sitting at $U = U_T$ and a point in S^4 while spanning x_4 and two directions in R^3 is the desired $k = 1$ domain wall in the gravity side. For small number of k, one naturally expects a collection of k $D2$-branes to be the corresponding object for the k-wall. Its tension is calculated straightforwardly by the DBI

action of $D2$-branes at $U = U_T$,

$$T_k = k\mu_2 \int dx_4\, e^{-\phi}\sqrt{g^*}\Big|_{U=U_T} = \frac{2\pi k}{(2\pi l_s)^3}\frac{2\pi}{g_s M_{KK}}\left(\frac{U_T}{R}\right)^{\frac{3}{2}} = \frac{32\pi^3}{27}kN\frac{T^3}{M_{KK}}\,. \quad (3.11)$$

This result is qualitatively different from both the weak-coupling calculations of pure SU(N) Yang-Mills theory [20, 21], and the recent weak/strong coupling calculations of $N = 4$ SYM by Armoni, Kumar and Ridgway [5], where the results have a common form of

$$T_k = (const)\cdot kN\frac{T^2}{\sqrt{g^2_{\rm YM}N}}\,, \quad (3.12)$$

for small $\frac{k}{N} \ll 1$.

One can understand the origin of the discrepancy as follows. The confinement- deconfinement phase transition in the Witten's background happens at a temperature T_c of order M_{KK} [17], and the above deconfinement background becomes relevant when T is much larger than M_{KK}. However, M_{KK} also serves as a compactification scale of x_4 below which we have a 4D gauge theory, while above which massive KK modes enter the dynamics and the theory becomes effectively 5-dimensional. The number of KK modes that would enter at the temperature T is roughly given by $\frac{T}{M_{KK}}$, and assuming each new degrees of freedom contributes to the tension of the domain wall, one can understand $\frac{T^3}{M_{KK}} = T^2\cdot\frac{T}{M_{KK}}$ behavior of our result for the tension formula. However, missing $\sqrt{g^2_{\rm YM}N}$ factor remains still puzzling. One possibility is that the mass of KK modes might become $\frac{M_{KK}}{\sqrt{g^2_{\rm YM}N}}$ at strong coupling, and the number of effective degrees of freedom at T might be $\frac{T}{M_{KK}}\sqrt{g^2_{\rm YM}N}$ instead of $\frac{T}{M_{KK}}$. Our result may be considered as pointing out this phenomenon. The above discussion indicates that the deconfinement geometry (3.5) that has been used in the previous literature has some limitation to be used as a gravity dual background of a 4D gauge theory in its deconfined phase.

We proceed by studying k-walls with k being large and comparable to N, motivated by a recent work of Armoni, Kumar and Ridgway in $AdS_5 \times S^5$ [5], and our method in this regard will be similar to theirs. The naive picture of k-wall as a simple collection of k $D2$-branes should break down when $k \sim N$ because k is Z_N-valued and moreover k is related to $(N - k)$ by charge conjugation and the tension must be invariant under this. Similar to the claim in ref. [5], we propose that k $D2$-branes sitting at a point in S^4 blow up into a single IIA $NS5$-brane wrapping $S^3 \subset S^4$ due to the background F_4 flux on S^4, and we confirm this picture by computing the k-wall tension and check the necessary properties.

To analyze more easily the Type IIA $NS5$-brane dynamics, with k $D2$-brane charges on its world-volume along x_4 and two spatial directions in R^3, say x_1 and x_2, we perform U-dualities of Type II theories in the following way. Note first that in the deconfined phase geometry (3.5), x_4 size remains finite in all region of the spacetime, contrary to the confined phase geometry where x_4 shrinks zero at $U = U_{KK}$. Hence, we are eligible to take T-duality along x_4 in the deconfined phase. We stress that this is not allowed in

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the original Witten's background with shrinking x_4, and one should take this operation applicable only to the present deconfined phase as a mere tool for calculating some physical quantities in an easier way. After T-duality, our $NS5$-brane becomes Type IIB $NS5$-brane because it wraps the x_4 direction, and the k $D2$-brane charges wrapping x_4 will transform to k $D1$-brane charges spanning now only x_1 and x_2. They are homogeneously distributed in x_4 and $S^3 \subset S^4$ of our $NS5$-brane world-volume. We next perform Type IIB S-duality in the system, such that we finally have a (Euclidean) $D5$-brane wrapping $S^3 \subset S^4$ and $x_{4,1,2}$, with k $F1$-string charges on its world-volume along $x_{1,2}$, homogeneously distributed in x_4 and $S^3 \subset S^4$. This can be studied by DBI plus Chern-Simons action.

The T-dualized background of (3.5) along x_4 looks as

$$\begin{aligned} d\tilde{s}^2 &= \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(f(U)dt_E^2 + dx_i^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}} (M_{KK}l_s)^2 d\tilde{x}_4^2 + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) , \\ F_5 &= \frac{(2\pi l_s)^3 N}{V_4} d\tilde{x}_4 \wedge \epsilon_4 , \qquad\qquad e^{\tilde{\phi}} = e^{\phi} \frac{1}{\sqrt{g_{44}}} = g_s (M_{KK} l_s) , \end{aligned} \tag{3.13}$$

where the parameter R and $f(U)$ are same as before, and more importantly $\tilde{x}_4$ has the same period $(2\pi l_s)$ as x_4. One should remember that the relevant rules of T-duality in ref. [22], that is, $\tilde{g}_{44} = \frac{1}{g_{44}}$ and $e^{\tilde{\phi}} = e^{\phi}\frac{1}{\sqrt{g_{44}}}$ must be applied in the coordinate with the fixed period $(2\pi l_s)$.[2] The resulting 5-form flux simply describes N $D3$-branes homogeneously distributed along $\tilde{x}_4$, and the dilation is constant as it should be in a $D3$ background. After a further S-duality, upon which $ds'^2 = e^{-\tilde{\phi}} ds^2$ and $e^{\phi'} = e^{-\tilde{\phi}}$, the final geometry is given by

$$\begin{aligned} ds'^2 &= \frac{1}{g_s (M_{KK} l_s)} \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(f(U)dt_E^2 + dx_i^2\right) + \frac{(M_{KK} l_s)}{g_s} \left(\frac{R}{U}\right)^{\frac{3}{2}} d\tilde{x}_4^2 \\ &\quad + \frac{1}{g_s (M_{KK} l_s)} \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) , \\ F_5 &= \frac{(2\pi l_s)^3 N}{V_4} d\tilde{x}_4 \wedge \epsilon_4 , \qquad\qquad e^{\phi'} = \frac{1}{g_s (M_{KK} l_s)} , \end{aligned} \tag{3.14}$$

and we consider a $D5$-brane wrapping $S^3 \subset S^4$ and $x_{4,1,2}$ with k $F1$ charges along $x_{1,2}$.

The (Euclidean) $D5$-brane action is (we omit primes in the above geometry)

$$S_E^{D5} = \mu_5 \int d^6\xi \, e^{-\phi} \sqrt{\det\left(g^* + 2\pi l_s^2 F\right)} - i\mu_5 \int C_4^{RR} \wedge 2\pi l_s^2 F , \tag{3.15}$$

with $\mu_p = (2\pi)^{-p} l_s^{-(p+1)}$ and we turn on the world-volume gauge flux $F = dA$ only along $x_{1,2}$ to represent k $F1$ charges. To find an expression of C_4^{RR} usable for our purpose that satisfies $dC_4^{RR} = F_5$ with F_5 given above, we introduce a polar angle $0 \le \theta < \pi$ on S^4 to write $d\Omega_4^2 = d\theta^2 + \sin^2\theta d\Omega_3^2$ and $\epsilon_4 = \sin^3\theta d\theta \wedge \epsilon_3$ with ϵ_n being the volume form of the unit S^n. Our $D5$-brane is wrapping S^3 at a constant θ whose value will be determined dynamically by a non-zero k $F1$ flux along $x_{1,2}$ on its world-volume. We choose the gauge such that C_4^{RR} is smooth around $\theta = 0$ to have

$$C_4^{RR} = \frac{(2\pi l_s)^3 N}{V_4} \left(\cos\theta - \frac{1}{3}\cos^3\theta - \frac{2}{3}\right) d\tilde{x}_4 \wedge \epsilon_3 . \tag{3.16}$$

[2] Moreover, $C_4^{\text{here}} = 4C_4^{\text{there}}$ and $C_3^{\text{here}} = \frac{3}{2} C_3^{\text{there}}$.

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With these data, it is rather straightforward to compute the above $D5$ action, except a subtle point that the k $F1$ charge along $x_{1,2}$ is represented by an imaginary $F_{12} \equiv iF$ [5]. One way of seeing this is to consider a Wick-rotated Lorenzian situation with x_1 being the time direction, where the $F1$ string charge would be unambiguously given by a real F_{12}. Upon going back to our Euclidean situation by the Wick-rotation of x_1, the $F1$ flux will transform to an imaginary value.

The resulting $D5$-brane action density on $x_{1,2}$ after integrating over S^3 and x_4 is

$$s_E^{D5} = \frac{N}{4}\sin^3\theta\sqrt{C\left(\frac{U}{U_T}\right)^3 - F^2} + \frac{3N}{4}\left(\cos\theta - \frac{1}{3}\cos^3\theta - \frac{2}{3}\right)F\,, \tag{3.17}$$

where (U,θ) is yet undetermined position of the $D5$-brane, and

$$C = \frac{4^5\pi^6 N^2 T^6}{3^6 M_{KK}^2} \quad . \tag{3.18}$$

The k $F1$ charge on its world-volume is now identified as a conserved flux

$$k = -\frac{\delta s_E^{D5}}{\delta F} = \frac{N}{4}\left(\frac{F\sin^3\theta}{\sqrt{C\left(\frac{U}{U_T}\right)^3 - F^2}} - 3\cos\theta + \cos^3\theta + 2\right)\,, \tag{3.19}$$

which can be solved for F in terms of (U,θ), and the effective Hamiltonian density one obtains after a Legendre transform with the conserved flux F becomes

$$h = s_E^{D5} + kF = \frac{N}{4}\sqrt{C\left(\frac{U}{U_T}\right)^3}\sqrt{\sin^6\theta + \left(3\cos\theta - \cos^3\theta - 2 + \frac{4k}{N}\right)^2}\,, \tag{3.20}$$

and we have to minimize this with respect to (U,θ) to find the k-wall tension in our framework. It is trivial to see that the $D5$-brane sits at the IR end $U = U_T$, while the size of S^3 inside S^4 given by the polar angle θ must be determined by solving the following equation

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{k}{N}\,. \tag{3.21}$$

As the function of θ in the left-hand side is a monotonic function in the range $[0,\pi]$ with values between $(0,1)$, one checks that $k \leftrightarrow (N-k)$ corresponds to $\theta \leftrightarrow (\pi - \theta)$ in the solution. With the solution of θ and $U = U_T$, the k-wall tension is finally given by

$$T_k = \frac{N}{4}\sqrt{C}\sin^2\theta = \frac{8\pi^3}{27}N^2\sin^2\theta\frac{T^3}{M_{KK}} = \frac{32\pi^3}{27}k(N-k)\frac{T^3}{M_{KK}}\,, \tag{3.22}$$

with the desired property of $T_k = T_{N-k}$. Note that we obtain the precise Casimir scaling $T_k \sim k(N-k)$, contrary to the $N=4$ SYM result in ref. [5]. For a small $\frac{k}{N} \ll 1$, one also confirms that the result reduces to the tension of k $D2$-branes (3.11),

$$T_k \approx \frac{32\pi^3}{27}kN\frac{T^3}{M_{KK}}\,, \qquad \frac{k}{N} \ll 1\,. \tag{3.23}$$

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4 Fate of Z_N vacua with fundamental flavors

In this section, we come to our second motivation, that is, studying what happens to the Z_N-vacua with a presence of fundamental flavors in a probe approximation, or equivalently in a quenched approximation, via gauge/gravity correspondence. We first consider the system of $D3/D7$-branes that is dual to $N = 4$ SYM perturbed by a small number of $N = 2$ fundamental hypermultiplets represented by N_f probe $D7$-branes in a N $D3$-brane background [6]. We are especially interested in the dependence on the mass of the flavors given by the asymptotic distance between $D7$ and $D3$-branes. We also calculate the energy lift of Z_N-vacua in the massless case analytically, and finally perform a similar analysis in the model by Sakai-Sugimoto [7] for a QCD-like theory with massless chiral quarks.

4.1 $D3/D7$ system

Although the appearance of discrete Z_N vacua in the deconefined phase geometry (2.8) of $N = 4$ SYM is a sub-leading effect of large N limit, one can still advocate a quenched approximation where one neglects back-reaction of the probe $D7$-branes to the $N = 4$ SYM dynamics including the mechanism of selecting Z_N vacua. We point out that this is just one type of approximation which is not solely based on the large N limit, because back-reactions of $D7$-branes may well be of the same sub-leading order of the previous mechanism of Z_N vacua in gravity. However, we feel that the present quenched approximation is a useful one to pursue due to its practical calculability and the possibility of its comparison to lattice QCD in the same quenched approximation.

Working in the quenched approximation in the gravity side means that we take the previous Z_N vacua of

$$\frac{1}{2\pi l_s^2}\int_D B^{NS} = \frac{2\pi k}{N}\,, \tag{4.1}$$

as a given background and consider the probe dynamics of $D7$-branes embedded in it. As we neglect possible back-reactions, questions regarding the fate of Z_N vacua become those of the probe $D7$-branes, such as whether the energy of k'th vacuum is lifted or not. We can address this question by studying the energy of the probe $D7$-brane in the k'th gravity vacuum (4.1).

The action of a probe $D7$-brane is

$$S_E^{D7} = \mu_7 \int d^8\xi\, e^{-\phi}\sqrt{\det\left(g^* + B^{NS*} + 2\pi l_s^2 F\right)}\,, \tag{4.2}$$

where the Chern-Simons term is irrelevant for our purpose. One should study the equations of motion of the embedding $X^M(\xi)$ and the world-volume gauge field $F = dA$ in the presence of the background B^{NS} in (4.1), and in general these two are coupled to each other, except the case of trivial vacuum $k = 0$ where one can turn off $F = 0$ consistently. This special case was analyzed in ref. [23] with varying hypermultiplet mass m_q, where they found a phase transition of meson-melting at $T_c \sim m_q$. Our problem is a more sophisticated version of theirs.

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To parameterize $D7$-brane embedding in the background of (2.8), it is convenient to introduce the rectangular coordinate $x^{4,5,6,7,8,9}$ such that r and S^5 coordinates are related to them by

$$dr^2 + r^2 d\Omega_5^2 = \sum_{i=4}^{9} (dx^i)^2 \,, \tag{4.3}$$

and in fact they are nothing but the original R^6 coordinates transverse to the $D3$-branes. Asymptotically at $r \to \infty$, our $D7$-brane spans four flat directions out of x^{4-9}, say x^{4-7}, being a point at $x^{8,9}$ with a distance $2\pi l_s^2 m_q$ from the center. This fixes the UV boundary condition specified by the mass of the fundamental flavor m_q. As r goes to the infrared region, the $D7$-brane would feel an attraction toward the horizon at $r = r_H$ and it bends. One easily finds that the spherical symmetry on x^{4-7} is a symmetry of the situation, and the $D7$-brane wraps S^3-sphere in x^{4-7} given a point in $x^{8,9}$. Moreover $D7$-brane would move only along one axis on the $x^{8,9}$-plane, and one can take $x^9 \equiv 0$ without loss of generality. Then, the embedding is simply given by a map between x^8 and the radius ρ of S^3 inside x^{4-7}. One notes that $r^2 = (x^8)^2 + (x^9)^2 + \rho^2$ and

$$dr^2 + r^2 d\Omega_5^2 = \left(dx^8\right)^2 + \left(dx^9\right)^2 + d\rho^2 + \rho^2 d\Omega_3^2 \,, \tag{4.4}$$

so that one can equally describe the embedding by a map between x^8 and r, which will be chosen from now on. We will choose the world-volume coordinates ξ^a by (t_E, x_i, r, Ω_3), and x^8 is a function of r specifying the embedding shape that is determined dynamically with the specified UV boundary condition at $r \to \infty$,

$$x^8(r \to \infty) = 2\pi l_s^2 m_q \,, \tag{4.5}$$

as we discuss in the above. The induced metric g^*_{ab} on the $D7$ world-volume is then

$$\begin{aligned} ds^2_{D7} = {} & \frac{r^2}{L^2}\left[\left(1 - \frac{\pi^4 T^4 L^8}{r^4}\right) dt_E^2 + dx_i^2\right] + \frac{L^2}{r^2}\left(r^2 - \left(x^8(r)\right)^2\right) d\Omega_3^2 \\ & + \frac{L^2}{r^2}\left[\frac{\left(r - x^8(r)\frac{dx^8(r)}{dr}\right)^2}{r^2 - \left(x^8(r)\right)^2} + \left(\frac{dx^8(r)}{dr}\right)^2 + \frac{\pi^4 T^4 L^8}{r^4 - \pi^4 T^4 L^8}\right] dr^2 \,, \end{aligned} \tag{4.6}$$

from which one easily computes the action density along $x_{1,2,3}$ after integrating over t_E and S^3,

$$s_E^{D7} = \frac{\beta}{2^6 \pi^5 g_s l_s^8} \int dr \, \left(r^2 - \left(x^8(r)\right)^2\right)^{\frac{3}{2}} \sqrt{A + \left(B_{0r}^{NS} + 2\pi l_s^2 F_{0r}\right)^2} \,, \tag{4.7}$$

with

$$A \equiv \left(1 - \frac{\pi^4 T^4 L^8}{r^4}\right)\left[\frac{\left(r - x^8(r)\frac{dx^8(r)}{dr}\right)^2}{r^2 - \left(x^8(r)\right)^2} + \left(\frac{dx^8(r)}{dr}\right)^2 + \frac{\pi^4 T^4 L^8}{r^4 - \pi^4 T^4 L^8}\right] \,, \tag{4.8}$$

where we turn on F_{0r} along (t_E, r) as a possible world-volume flux in response to the given background B_{0r}^{NS} with

$$\frac{\beta}{2\pi l_s^2} \int_{r_H}^{\infty} dr \, B_{0r}^{NS} = \frac{2\pi k}{N} \,. \tag{4.9}$$

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We are assuming symmetry along the thermal circle t_E without loss of generality.

The equation of motion of the gauge field is simple to solve; $\frac{\delta s_E^{D7}}{\delta F_{0r}}$ is a constant of motion,

$$\frac{\delta s_E^{D7}}{\delta F_{0r}} \sim \frac{\left(r^2 - \left(x^8(r)\right)^2\right)^{\frac{3}{2}} \left(B_{0r}^{NS} + 2\pi l_s^2 F_{0r}\right)}{\sqrt{A + \left(B_{0r}^{NS} + 2\pi l_s^2 F_{0r}\right)^2}} \equiv r_H^3 C \,, \tag{4.10}$$

with some constant C and we factored out r_H^3 for later convenience. One can solve the above for $\left(B_{0r}^{NS} + 2\pi l_s^2 F_{0r}\right)$, and the remaining dynamics of the $D7$ embedding $x^8(r)$ will then be described by the Routhian,

$$R_E^{D7} = s_E^{D7} - \int dr \left(\frac{1}{2\pi l_s^2} B_{0r}^{NS} + F_{0r}\right) \frac{\delta s_E^{D7}}{\delta F_{0r}} = \frac{\beta}{2^6 \pi^5 g_s l_s^8} \int dr \sqrt{A} \sqrt{\left(r^2 - \left(x^8(r)\right)^2\right)^3 - r_H^6 C^2}, \tag{4.11}$$

given a value of C. The quest is to scan possible range of C and the resulting $D7$-brane embedding shape from solving the above R_E^{D7}, to minimize the original action density s_E^{D7}. Once the final configuration is found, one can compute its energy density from s_E^{D7} to address the question of energy lift of Z_N vacua. The necessary numerical analysis, as one varies the quark mass m_q, is beyond the scope of the present work. Identifying a phase transition would be an interesting future direction.

However, we make several important remarks. Note that in our coordinate parametrization of $D7$ embedding, there is an inequality

$$x^8(r) \leq r, \tag{4.12}$$

and the equality $x^8(r) = r$ happens precisely when S^3 shrinks to zero, where the $D7$-brane stops going into a lower r, that is, it is the end point of the $D7$ embedding in the radial direction r. If this happens at $r = r_0 > r_H$ in the solution, the $D7$-brane doesn't touch the horizon, and we are in the phase of un-melted mesons in the gauge theory side. It is important to observe that in this phase the $D7$-brane covers only part of the (t_E, r) cigar D of the background (2.8), that is, a cylinder topology of $[r_0, \infty] \times t_E$ inside the cigar D. Looking back the expression (4.10), one readily finds that the constant of motion C must be zero in this phase considering the point $r = r_0$, and this in turn says that

$$\frac{1}{2\pi l_s^2} B_{0r}^{NS} + F_{0r} \equiv 0 \,, \tag{4.13}$$

in all region of the $D7$-brane world-volume. Then the analysis of the embedding shape and the resulting energy density from s_E^{D7} becomes precisely equal to the case of trivial vacuum $k = 0$ or without B^{NS}/F fluxes at all. One concludes that Z_N-vacua are not lifted and persist to exist in this phase in quenched approximation. One interprets that the world-volume flux F nullifies the background B^{NS} flux to minimize s_E^{D7}. In the gauge theory point of view, as the world-volume gauge symmetry on the $D7$-brane is the flavor global symmetry in the gauge theory side, this points out an interesting phenomenon of a dynamically generated monodromy of flavor symmetry along the thermal circle that counteracts the existing Z_N Wilson line in the Z_N-vacua, to make the Z_N-vacua viable

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even in the presence of fundamental flavors. One mathematical remark is that (4.13) is possible precisely because the topology of $D7$-brane in (t_E, r) is cylindrical, so that there is no topological restriction of its $\int F$ value.

In the other phase where $x^8(r) < r$ remains true until it hits the horizon $r = r_H$, the $D7$-brane meets the horizon and the mesons are melted. A crucial point in this phase is that the $D7$-brane now wraps the whole cigar D of (t_E, r) with topology of a two-plane, and there is an important topological restriction

$$\int_D F = 2\pi m\,, \qquad m \in Z\,. \tag{4.14}$$

Suppose one tries to put $C = 0$, or equivalently $\frac{1}{2\pi l_s^2} B_{0r}^{NS} + F_{0r} \equiv 0$, to minimize the action s_E^{D7} as before, then

$$\int_D F = -\frac{1}{2\pi l_s^2} \int_D B^{NS} = -\frac{2\pi k}{N} \notin 2\pi Z\,, \tag{4.15}$$

except the trivial vacuum $k = 0$. In other words, in the k'th vacuum it is not possible to completely nullify the Z_N Wilson line B^{NS} due to a topological restriction, and one has a non-vanishing $C \neq 0$ or $\frac{1}{2\pi l_s^2} B_{0r}^{NS} + F_{0r} \neq 0$ in the non-trivial Z_N vacua. For each integer m, C must be determined to satisfy (4.14), and one should choose m that minimizes the energy s_E^{D7}. Looking at the expression (4.7) of s_E^{D7}, one sees that the resulting energy is always higher than the trivial vacuum $k = 0$ where $\frac{1}{2\pi l_s^2} B_{0r}^{NS} + F_{0r} \equiv 0$, and we conclude that the Z_N vacua are lifted in the phase of melted mesons.

The question at which phase the system finds its solution given the UV boundary condition of the flavor mass m_q should be addressed numerically, but for a sufficiently large $\frac{m_q}{T} \gg 1$ it is natural to expect the former phase where the mesons are un-melted and the Z_N vacua survive, while in the extreme case of $m_q = 0$ it is obvious that $x^8(r) \equiv 0$ and we are in the latter phase of melted mesons with Z_N-vacua lifted. One naturally expect a phase transition to happen at some point $T_c \sim m_q$. An interesting question is whether the critical temperature T_c depends on k or not because of the additional contribution from the B^{NS}/F fluxes to s_E^{D7}, whose answer is beyond the present paper.

In the simplest case of $m_q = 0$ where one can consistently set $x^8(r) \equiv 0$ to be in the melted-meson phase, it is possible to compute the energy lift of the Z_N-vacua analytically. Note that we have $A = 1$ in (4.7) with $x^8(r) \equiv 0$, and the equation of motion for the gauge field (4.10) simplifies as

$$\frac{r^3 \left(B_{0r}^{NS} + 2\pi l_s^2 F_{0r}\right)}{\sqrt{1 + \left(B_{0r}^{NS} + 2\pi l_s^2 F_{0r}\right)^2}} \equiv r_H^3 C\,, \tag{4.16}$$

which can be solved for F,

$$F_{0r} = -\frac{1}{2\pi l_s^2} B_{0r} + \frac{C}{2\pi l_s^2 \sqrt{\left(\frac{r}{r_H}\right)^6 - C^2}}\,. \tag{4.17}$$

As discussed before, C can not take any continuous values, but is restricted to only discrete values to satisfy the topological constraint (4.14). Explicitly, integrating both sides of (4.17)

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over D, one has

$$2\pi m = -\frac{2\pi k}{N} + \frac{C\beta}{2\pi l_s^2}\int_{r_H}^{\infty} dr\, \frac{1}{\sqrt{\left(\frac{r}{r_H}\right)^6 - C^2}} = -\frac{2\pi k}{N} + \frac{C\beta r_H}{2\pi l_s^2}\int_1^{\infty} d\hat{r}\, \frac{1}{\sqrt{\hat{r}^6 - C^2}}\,, \quad (4.18)$$

so that given (m,k), the C is determined by

$$C\int_1^{\infty} d\hat{r}\, \frac{1}{\sqrt{\hat{r}^6 - C^2}} = \frac{C}{2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},C^2\right) = 2\pi\left(m + \frac{k}{N}\right)\frac{1}{\sqrt{\pi g_s N}}\,, \quad (4.19)$$

where we have used $r_H = \pi T L^2 = \pi T\,(4\pi g_s N)^{\frac{1}{2}}\, l_s^2$. Once C is determined as a function of (m,k), it is straightforward to calculate the energy density s_E^{D7},

$$\begin{aligned} s_E^{D7} - s_E^{D7}\big|_{k=0} &= \frac{\beta r_H^4}{2^6\pi^5 g_s l_s^8}\int_1^{\infty} d\hat{r}\, \left(\frac{\hat{r}^6}{\sqrt{\hat{r}^6 - C^2}} - \hat{r}^3\right) \\ &= \frac{\pi g_s N^2 T^3}{4}\int_1^{\infty} d\hat{r}\, \left(\frac{\hat{r}^6}{\sqrt{\hat{r}^6 - C^2}} - \hat{r}^3\right), \end{aligned} \quad (4.20)$$

where we have subtracted the value of the trivial vacuum $k = 0$ to see the energy lift of the (m,k) state. For a fixed k, one should choose m that minimizes the energy to finally find the energy lift of the k-vacuum,

$$\Delta\epsilon_k = \min_{m\in Z}\left(s_E^{D7}\big|_{(m,k)} - s_E^{D7}\big|_{k=0}\right). \quad (4.21)$$

From (4.20), one has to minimize C^2, and an inspection of (4.19) shows that this is achieved by the smallest value of $|m + \frac{k}{N}|$. Therefore, our result is invariant under $k \to k+N$ and $k \to (N-k)$, which are the desired properties due to Z_N nature and charge conjugation. For low lying k-vacua with $\frac{k}{N} \ll 1$, we have $m = 0$ to minimize the energy, and an easy calculation gives us

$$\Delta\epsilon_k \approx \pi^2\frac{k^2}{N}T^3\,, \quad \frac{k}{N} \ll 1\,. \quad (4.22)$$

One also finds that $\Delta\epsilon_k$ has a discontinuous slope at $\frac{k}{N} = \frac{1}{2}$.

4.2 The model of Sakai and Sugimoto

The Sakai-Sugimoto model is obtained by introducing probe $D8/\bar{D}8$ branes in the Witten's $D4$-brane background, that span $S^4 \times R^{3,1} \times U$ and are point-like in x_4-direction. In the original weak-coupling $D4/D8/\bar{D}8$-brane picture, one finds left(right)-handed chiral quarks from $D4-D8(D4-\bar{D}8)$ string spectrum, and at low energy the resulting 4D gauge theory flows precisely to QCD with massless chiral flavor quarks. One therefore expects the gravity picture to capture some of the interesting dynamics of this semi-realistic gauge theory, at least for low enough energy regime. In the confined geometry (3.1) where $(x_4, U \geq U_{KK})$ makes a cigar shape with vanishing x_4 at $U = U_{KK}$, the $D8/\bar{D}8$-brane pair must join with each other at IR, realizing spontaneous chiral symmetry breaking in an intuitively geometric way. Our current interest is however on the opposite case of the deconfined

geometry (3.5), where x_4 remains finite everywhere, and we instead have a cigar shape D with $(t_E, U \geq U_T)$. In this case, each $D8$ and $\bar{D}8$-brane separately meets the horizon at $U = U_T$ and their world-volume embedding is simply parameterized by a constant x_4 position. We will consider a single pair of $D8/\bar{D}8$-branes for simplicity.

As we consider $D8/\bar{D}8$-branes in the Z_N-vacua of the background,

$$\frac{1}{2\pi l_s^2}\int_D B^{NS} = \frac{\beta}{2\pi l_s^2}\int_{U_T}^{\infty} dU\, B^{NS}_{0U} = \frac{2\pi k}{N}\,, \tag{4.23}$$

one can easily find that the embedding shape can still be consistently given by a constant x_4 from the equation of motion, and we will take this without further detail. One can also be convinced by translational symmetry along x_4. Because the Chern-Simons term of the $D8/\bar{D}8$ doesn't play any role here, the actions of $D8$ and $\bar{D}8$ are identical, and the total action is two times the DBI action of the $D8$-brane,

$$S_E = 2\mu_8 \int d^9\xi\, e^{-\phi}\sqrt{\det\left(g^* + B^{NS*} + 2\pi l_s^2 F\right)}\,, \tag{4.24}$$

where we choose the world-volume coordinates ξ^a to be (t_E, x_i, U, Ω_4). It is straightforward to compute the above, turning on a possible world-volume gauge field F_{0U} along the cigar D of (t_E, U). After integrating over $S^4 \times t_E$, the action density in the spatial R^3-direction becomes

$$s_E = \frac{N^{\frac{1}{2}}}{3\cdot 2^4 \pi^{\frac{11}{2}} g_s^{\frac{1}{2}} l_s^{\frac{15}{2}} T}\int_{U_T}^{\infty} dU\, U^{\frac{5}{2}}\sqrt{1+\left(B^{NS}_{0U} + 2\pi l_s^2 F_{0U}\right)^2}\,. \tag{4.25}$$

The equation of motion for F_{0U} is simply $\frac{\delta s_E}{\delta F_{0U}} = \text{const.}$, which we parameterize by

$$\frac{U^{\frac{5}{2}}\left(B^{NS}_{0U} + 2\pi l_s^2 F_{0U}\right)}{\sqrt{1+\left(B^{NS}_{0U} + 2\pi l_s^2 F_{0U}\right)^2}} \equiv U_T^{\frac{5}{2}} C\,, \tag{4.26}$$

with a dimensionless constant C, and from this one easily solves for F_{0U} as

$$F_{0U} = -\frac{1}{2\pi l_s^2}B_{0U} + \frac{1}{2\pi l_s^2}\frac{C}{\sqrt{\left(\frac{U}{U_T}\right)^5 - C^2}}\,. \tag{4.27}$$

As in the previous case of $D3/D7$ system, an important point is a topological restriction

$$\int_D F = \beta\int_{U_T}^{\infty} dU\, F_{0U} = 2\pi m\,, \quad m \in Z\,, \tag{4.28}$$

and this determines the constant C in terms of given (k, m) as follows,

$$\begin{aligned} C\int_1^{\infty} d\hat{U}\frac{1}{\sqrt{\hat{U}^5 - C^2}} &= \frac{2C}{3}\,{}_2F_1\left(\frac{3}{10}, \frac{1}{2}, \frac{13}{10}, C^2\right) \\ &= \left(m + \frac{k}{N}\right)\frac{4\pi^2 l_s^2}{\beta U_T} = \left(m + \frac{k}{N}\right)\frac{9}{4\pi g_s N l_s T}\,, \end{aligned} \tag{4.29}$$

where ${}_2F_1$ is the hypergeometric function. Once C is given, one can easily calculate the difference in the action density s_E from the $k=0$ vacuum where $\left(B_{0U}^{NS}+2\pi l_s^2 F_{0U}\right)=0$, and after sorting out the coefficients, the final answer is

$$s_E - s_E\Big|_{k=0} = \frac{2^{10}\pi^5 g_s^3 N^4 l_s^3 T^6}{3}\int_1^\infty d\hat{U}\,\hat{U}^{\frac{5}{2}}\left(\sqrt{\frac{\hat{U}^5}{\hat{U}^5-C^2}}-1\right)\,. \tag{4.30}$$

For a given k-vacuum, one should find the integer m such that it minimizes the above result, and that is the final energy lift of the k'th vacuum $\Delta\epsilon_k$. Because C^2 is a monotonic increasing function of $\left|m+\frac{k}{N}\right|$, one should take the smallest possible value of $\left|m+\frac{k}{N}\right|$, and this again gives us $\Delta\epsilon_k=\Delta\epsilon_{k+N}$ and $\Delta\epsilon_k=\Delta\epsilon_{N-k}$. The slope is discontinuous in the middle $\frac{k}{N}=\frac{1}{2}$. For small $\frac{k}{N}\ll 1$, one has $m=0$ and one can easily find that

$$C\approx\frac{27}{8\pi}\frac{1}{g_s N l_s T}\frac{k}{N}\,,\quad \frac{k}{N}\ll 1\,, \tag{4.31}$$

and

$$\Delta\epsilon_k\approx 6^4\pi^3 g_s l_s k^2 T^4 = 2^3 3^4\pi^2\left(g_{\rm YM}^2 N\right)\frac{k^2}{N}\frac{T^4}{M_{KK}}\,,\quad \frac{k}{N}\ll 1\,, \tag{4.32}$$

where we have used $g_s l_s=\frac{g_{\rm YM}^2}{2\pi M_{KK}}$. This is qualitatively different from the case of $D3/D7$ system (4.22).

5 Conclusion

In this work, we first study k-walls with $k\sim N$ in the Witten's $D4$-brane background of pure SU(N) Yang-Mills theory in large N limit, largely motivated by a recent work in $AdS_5\times S^5$ by Armoni, Kumar and Ridgway [5]. We propose and check consistency of the picture that k (Euclidean) $D2$-branes blow up into a $NS5$-brane wrapping S^3 inside S^4 of the background via Myers effect. We compute the tension of k-wall by performing suitable U-duality to transform the $NS5$-brane into a $D5$-brane. Our result is a precise Casimir scaling behavior of the k-wall tension $T_k\sim k(N-k)$. We note that the same Casimir scaling was also obtained for the k-quark flux tube tension in ref. [24].

Our second subject is to consider fate of Z_N-vacua in the presence of fundamental flavors in quenched approximation via gauge/gravity correspondence. In our study of $D3/D7$ system, we point out an interesting phenomenon of phase transition as one varies $\frac{m_q}{T}$, which separates the phase of un-melted mesons with Z_N-vacua survived and the phase of melted mesons with Z_N-vacua lifted. An interesting question is whether the critical temperature depends on k, which should be addressed by a numerical analysis in the future.

In the special case of $m_q=0$, we calculate the energy lift of k'th vacua analytically. Our result is consistent with the Z_N-nature and the charge conjugation where $k\to(N-k)$. We also find a tower of stable states parameterized by integers m above the lowest energy state. We do the similar analysis in the Sakai-Sugimoto model for an approximate QCD-like theory.

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